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Hitoshi Mada [a]

[a] Department of Electronic Engineering, Faculty of Technology, Tokyo University of Agriculture and Technology, Koganei, Tokyo, 184, JAPAN

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WALL EFFECT OF FREDERIKS TRANSITION IN NEMATIC LIQUID CRYSTALS

HITOSHI MADA

Department of Electronic Engineering,
Faculty of Technology,
Tokyo University of Agriculture and Technology,
Koganei, Tokyo 184, JAPAN.

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According to the elastic theory, the threshold voltage of the liquid crystal cell does not depend on the layer thickness. But the thickness dependence was shown experimentally for the homeotropic aligned methoxybenzyliden-p-buthylaniline. We discuss the Frederiks transition for the weak anchoring liquid crystal cell. Taking into account the surface interaction energy, the calculated result of the threshold voltage agrees well with the experimental results.

Frederiks and Zwetkoff[1)] showed there is a threshold voltage to reorient the liquid crystl for the homogeneous alignment. According to Gruler and Meier[2)], the threshold voltage can be expressed as $\pi(k_{11}/\varepsilon_0\Delta\varepsilon)^{1/2}$; where k_{11} is Frank constant, ε_0 and $\Delta\varepsilon$ are the dielectric constant in vacuum and the dielectric anisotropy of the liquid crystal, respectively. This expression was derived in the case that the liquid crystal is strongly anchored and can not move at the surface. The threshold voltage in the above expression does not depend on the cell thickness.

Tanamachi[3)] however showed experimentally that the threshold voltage does depend on the cell thickness for a homeotropically aligned methoxybenzyliden-p-buthylaniline (MBBA). This fact can not be explained theoretically with strong anchoring. In this letter, we will discuss the Frederiks transition for the case of the weak anchoring.

Equilibrium state of the director is given by the following well-known torque equation for the homogeneously aligned nematic liquid crystals with the electric field.

$$(1-k\sin^2\theta)\frac{d^2\theta}{dz^2} - k\sin\theta\cos\theta(\frac{d\theta}{dz})^2 + \frac{\varepsilon\sin\theta\cos\theta D_z^2}{k_{11}\varepsilon_0\varepsilon_\perp(1+\varepsilon\sin^2\theta)^2} = 0, \quad (1)$$

where $k=(k_{11}-k_{33})/k_{11}$, $\varepsilon=\Delta\varepsilon/\varepsilon_\perp$, and D_z is the z-component of the electrical displacement. When the liquid crystal is anchored strongly at the surface (i.e. $\theta\equiv0$), the threshold voltage is derived as $\pi(k_{11}/\varepsilon_0\Delta\varepsilon)^{1/2}$ from eq.(1)[3]. For the liquid crystal having a negative dielectric anisotropy, such as MBBA, k_{11} is replaces by k_{33} and $\Delta\varepsilon$ is taken an absolute value. The surface interaction energy plays an important roll for the weak anchoring case. Therefore we must consider the surface torque equation[5]:

$$\sin\psi\cos\psi[\overline{k}\{(\overline{\frac{d\theta}{dz}})^2+\sin\psi\cos\psi\overline{\frac{d^2\theta}{dz^2}}\}-\frac{\Delta\pi}{k_{11}}]+(1-k\sin^2\theta_0)\overline{\frac{d\theta}{dz}} = 0, \quad (2)$$

where θ_0 is the director tilt angle at the surface, $\psi=\theta_0-\phi$, and ϕ is the tilt angle of the easy axis measuring from the surface. The quantity $\overline{k}$ is $(\sin^2\theta\overline{k}_1+\overline{k}_2)/k_{11}$; $\overline{k}_1$ and $\overline{k}_2$ are the surface elastic constant. The interaction energy is expressed by the quantity $\Delta\pi$ which means the anisotropy of an interaction energy between the liquid crystal and the solid surface[4]. The bar in the equation stands for the quantity at the surface. The eqs.(1) and (2) give the equilibrium state of the weak anchored nematic liquid crystal cell.

When there is no applied voltage, the equilibrium state is realized as $\psi=0$, $d\theta/dz=0$, and $d^2\theta/dz^2=0$, i.e. the director tilts everywhere by ϕ. When the voltage is applied to the liquid crystal, $d\theta/dz\neq0$ and $d^2\theta/dz^2\neq0$, i.e. the liquid crystal deforms. If the easy axis lays on the surface ($\phi=0$), the undeformed state is also a stable state even when the voltage is applied as can be derived from eqs.(1) and (2). In other words, the threshold voltage exists only when $\phi=0$[5].

Starting from the undeformed state, the deformation is small vicinity the threshold voltage. We can write the deformed director n(z) to be

$$n(z) = n_0 + \delta n(z), \tag{3}$$

where n_0 is the undeformed state. The second term $\delta n(z)$ is perpendicular to n_0 and parallel to the electric field E (see Fig.1). The deformation energy F_d is obtained from Frank theory:

$$F_d = \frac{1}{2}k_{11}\left(\frac{\partial \delta n}{\partial z}\right)^2. \tag{4}$$

The electric energy gives a contribution

$$F_e = -\frac{1}{2}\varepsilon_0 \Delta\varepsilon E^2 \delta n^2. \tag{5}$$

We consider the deformation at the slightly higher voltage from the threshold voltage as shown in Fig.2. It is convenient to

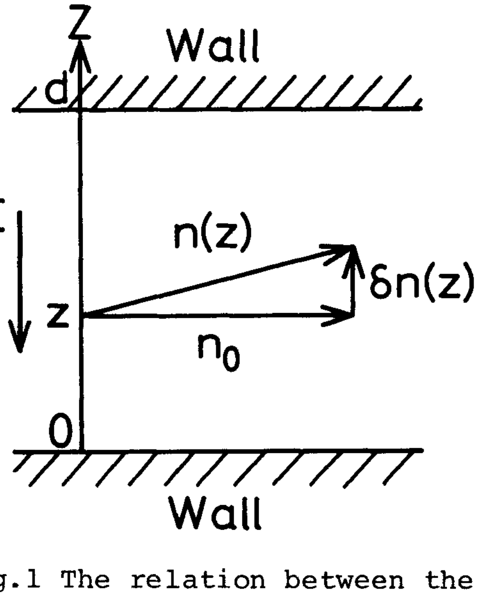

Fig.1 The relation between the undeformed director n_0 and the deformed director n(z).

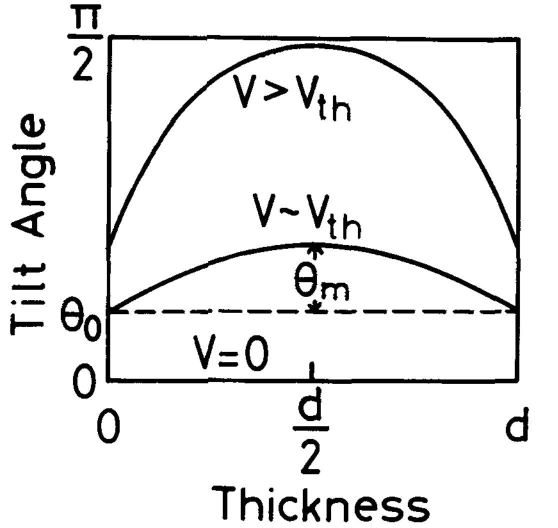

Fig.2 The director deformation of the weak anchoring liquid crystal cell.

analyse $\delta n(z)$ in a Fourier series.

$$\delta n(z) = \sin\theta_0 + \sum_q \delta n_q \sin qz, \tag{6}$$

where $q=\nu\pi/d$, ν is positive integer, and d is cell thickness. Inserting this form of $\delta n(z)$ into eqs.(4) and (5), and integrating once over the thickness, the free energy per unit area of slab is obtained

$$F = \frac{d}{4}\sum_q \delta n_q^2 (k_{11}q^2 - \varepsilon_0 \Delta\varepsilon E^2) - \frac{d}{4}\sin^2\theta_0 \varepsilon_0 \Delta\varepsilon E^2. \tag{7}$$

Since the undeformed state is stable, the free energy must be positive for all value of the parameter δn_q. The minimum electric field is given by $q=\pi/d$ ($\nu=1$) corresponding to a deformation of half-wavelength d biased by θ_0 as shown in Fig.2. We can put $E=V/d$ at $V=V_{th}$, the threshold voltage is then

$$V_{th} = \left(\frac{\delta n_1^2}{\delta n_1^2+\sin^2\theta_0}\right)^{\frac{1}{2}}\left(\frac{k_{11}}{\varepsilon_0\Delta\varepsilon}\right)^{\frac{1}{2}}. \quad (8)$$

From Fig.2, $\delta n_1^2=\sin^2\theta_m$. If we put the V_{th} as V_s for the strong anchoring case, the eq.(8) is rewritten:

$$V_{th} = \left(\frac{r}{1+r}\right)^{\frac{1}{2}}V_s,$$

where $r=\sin^2\theta_0/\sin^2\theta_m$. The ratio r must be positive, thus $V_{th}<V_s$

Next, we will estimate the ratio r from eqs.(1) and (2). Since the deformation is small around the threshold voltage, eq. (6) can be written as

$$\theta(z) = \theta_0+\theta_m\sin\frac{\pi}{d}z. \quad (10)$$

Substituting this form of θ into eqs.(1) and (2), and we assumed $\theta_0<<1$ and $\theta_m<<1$. Then one obtains the following expression from eqs.(1) and (2) by neglecting the higher order in θ_0 and θ_m. We also assumed r<1 for simplicity.

$$r = \frac{k_{33}\pi d}{(1-k)\Delta\pi d^2 - 2k_{11}k\pi d + k_{11}\bar{k}\pi^2} \quad (11)$$

If the anisotropy of the interaction energy is infinite, r becomes zero (i.e. $V_{th}=V_s$). This situation corresponds to the strong anchoring. In the case of finite Δπ, the threshold voltage is smaller than V_s. The cell thickness becomes infinite or zero, the ratio r reduces zero so one uses the thick cell, V_{th} is as same as the strong anchoring case eventhough the interaction between the liquid crystal and the surface is weak.

Figure 3 shows the comparison between the experimental results[3] and our estimation. The solid circles are the experimental results using MBBA, and initial alignment was homeotropic. The solid line is our result, the physical parameters used in the calculation are: $\Delta\varepsilon=-0.47$, $k_{11}=5.7\times10^{-12}$N, $k_{33}=7.0\times10^{-12}$N, $\bar{k}=3.2\times10^{-5}$, and $\Delta\pi=1.1\times10^{-5}$J/m[6]. The calculated result agrees well

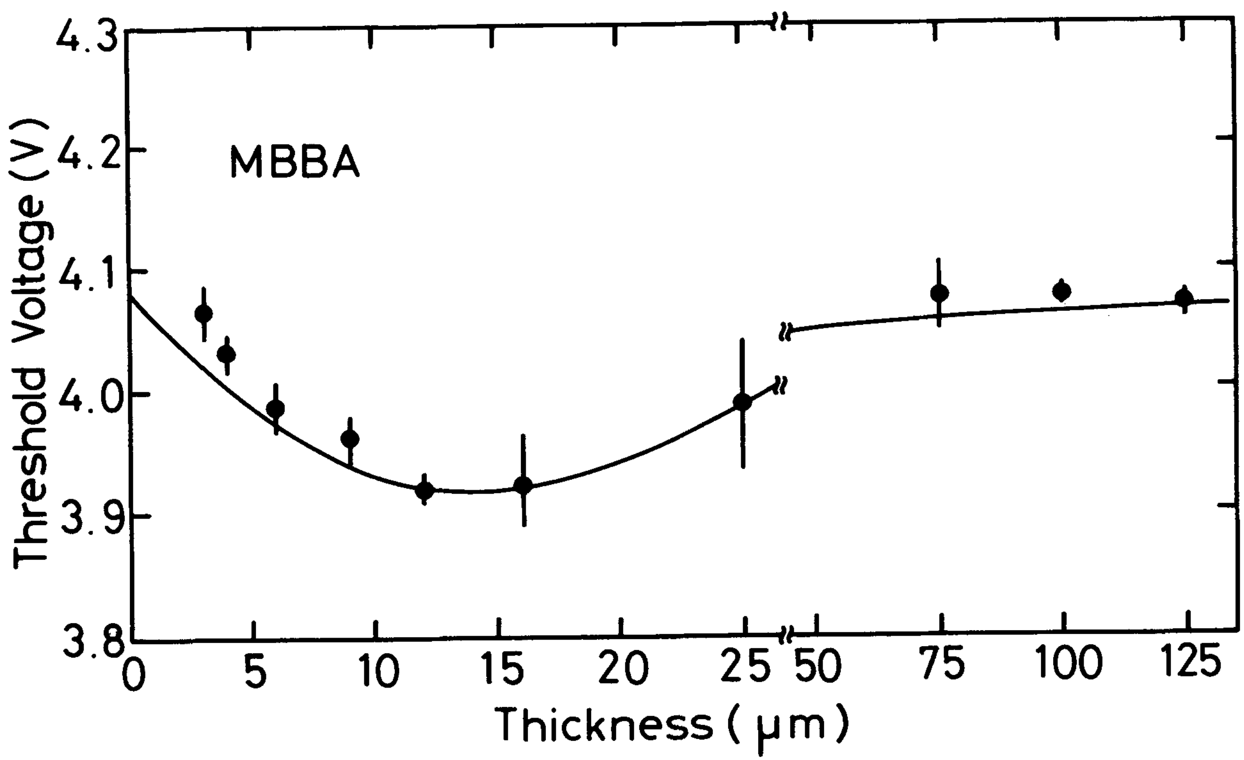

Fig.3 Thickness dependence of the threshold voltage. The solid line and the solid circles are the calculating result and the experimental results, respectively.

with the experimental results except the region of small d. Such a discrepancy may occur from the approximations when we derived the eq.(11). Actually, the ratio r is of the order of unity when the cell thickness is 2μm. The assumption $r<1$ is not valid for a very thin layer.

In conclusion, Frederiks transition voltage is modified by the magnitude of the surface interaction energy. For weak anchoring, the threshold voltage is smaller than that of the strong anchoring case; V_{th} is a function of the thickness of liquid crystal layer. Thickness dependence of V_{th} agrees with the experimental results. More precise solution will be necessary for thin samples.

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